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SUMMARY

The paper presents a theoretical analysis of the time variation of strain gradients in a tensile specimen of rate-dependent material, the analysis being based on the assumption that the strain rate in the material is a function of the local values of stress and strain. The theory is used to determine the criterion for the growth of strain gradients, and it is shown that for a given material there exists a region of the stress-strain plane in which these gradients increase indefinitely with time. The theory is applied to materials with specific types of rate-dependence, and the results are related to experimental data obtained at constant rates of strain.

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ACKNOWLEDGEMENTS

I. INTRODUCTION

It is well-known that in a static tension test non-uniform plastic flow occurs when the maximum load is reached, at which point the 'geometrical softening' of the specimen due to reduction of cross-sectional area becomes equal to the rate of work hardening. This condition is easily defined for a material with a rate-independent stress-strain relation, but cannot be applied to a rate-dependent material, which has no well-defined work-hardening rate.

Experimental data concerning the effect of strain rate on the fracture elongation of materials are confusing, since in some tests the elongation is found to increase at high rates while in others it is found to decrease. The fracture elongation is determined partly by the strain at which necking starts (the necking strain), and partly by the speed with which the neck forms during continued extension of the specimen. It is believed that both of these factors depend, in general, on the rate sensitivity of the material.

The purpose of the present paper is to put forward a theoretical analysis of the development of non-uniform plastic flow in tensile tests of rate-dependent materials, to discuss the implications of this analysis, and to relate these implications to the results of experiments on particular materials.

II. THE DEVELOPMENT OF STRAIN GRADIENTS IN A TENSION SPECIMEN

We consider a tension specimen of initial cross-sectional area A_o, which is assumed to be a function of the Lagrangian coordinate x, measured along the axis of the specimen; the variations in A_o are assumed small. The specimen is subjected to an axial force T, which increases in an arbitrary manner with time t. The cross-sectional area at time t is A(x,t), and the (engineering) strain is ε(x,t).

The rate-dependence of the specimen material is assumed to be governed by the equation

$$\dot{\varepsilon}_p = g(\sigma, \varepsilon_p), \tag{1}$$

where ε_p is the plastic strain. This equation is expected from dislocation theory, since the plastic strain rate is determined by the density and mean velocity of moving dislocations, each of which is, to a first approximation at least, related to the instantaneous stress and plastic strain. Equation (1) is similar in form to equations which have been postulated in theoretical studies of plastic wave propagation in rate-dependent materials (Sokolovsky 1948, Malvern 1951).

Since the onset of plastic instability in ductile materials occurs at strains which are large compared to elastic strains, the latter will be neglected and (1) rewritten as

$$\dot{\varepsilon} = g(\sigma, \varepsilon). \tag{2}$$

It should be noted that in (1) and (2) σ is the true local stress T/A. Since elastic strains are neglected, the volumetric strain is zero, which

requires that

$$A(1+\epsilon) = A_o, \tag{3}$$

and hence (2) may be written

$$\dot{\epsilon} = g[\frac{T}{A_o}(1+\epsilon),\epsilon]. \tag{4}$$

The strain gradient at (x,t) is defined by

$$\lambda = \frac{1}{1+\epsilon}\frac{\partial\epsilon}{\partial x} \tag{5}$$

so that

$$\frac{\partial\lambda}{\partial t} = \frac{1}{1+\epsilon}\frac{\partial\dot{\epsilon}}{\partial x} - \frac{\dot{\epsilon}}{(1+\epsilon)^2}\frac{\partial\epsilon}{\partial x}. \tag{6}$$

Substituting (4) into (6), we obtain

$$\frac{\partial\lambda}{\partial t} = \frac{1}{1+\epsilon}[\frac{T}{A_o}\frac{\partial\epsilon}{\partial x} - \frac{T(1+\epsilon)}{A_o^2}\frac{dA_o}{dx}]\frac{\partial g}{\partial\sigma} + \frac{\partial\epsilon}{\partial x}\frac{\partial g}{\partial\epsilon} - \frac{\dot{\epsilon}}{(1+\epsilon)^2}\frac{\partial\epsilon}{\partial x}$$

or

$$\frac{\partial\lambda}{\partial t} + P\lambda = Q, \tag{7}$$

where

$$P \equiv \frac{\dot{\epsilon}}{1+\epsilon} - \frac{\partial g}{\partial\epsilon} - \frac{\sigma}{1+\epsilon}\frac{\partial g}{\partial\sigma} \tag{8}$$

and

$$Q \equiv -\frac{\sigma}{1+\epsilon}\frac{\partial g}{\partial\sigma}\frac{1}{A_o}\frac{dA_o}{dx}. \tag{9}$$

It follows from (7) that $|\lambda|$ increases indefinitely with t unless $P > 0$, i.e.

$$\frac{\partial g}{\partial\epsilon} + \frac{\sigma}{1+\epsilon}\frac{\partial g}{\partial\sigma} < \frac{\dot{\epsilon}}{1+\epsilon}. \tag{10}$$

The inequality (10) is satisfied in some region of the (σ,ϵ) plane, which thus defines the conditions under which the strain distribution along the specimen remains approximately uniform as straining continues.

It may be more convenient to rewrite (10) in terms of the nominal or engineering stress $\sigma_n = T/A_o = \sigma/(1+\epsilon)$. Thus (2) becomes

$$\dot{\epsilon} = g_1(\sigma_n,\epsilon) \tag{11}$$

so that

$$\frac{\partial g}{\partial \sigma} = \frac{\partial g_1}{\partial \sigma_n}\frac{\partial \sigma_n}{\partial \sigma} = \frac{1}{1+\epsilon}\frac{\partial g_1}{\partial \sigma_n} \tag{12}$$

and

$$\frac{\partial g}{\partial \epsilon} = \frac{\partial g_1}{\partial \sigma_n}\frac{\partial \sigma_n}{\partial \epsilon} + \frac{\partial g_1}{\partial \epsilon} = -\frac{\sigma}{(1+\epsilon)^2}\frac{\partial g_1}{\partial \sigma_n} + \frac{\partial g_1}{\partial \epsilon}. \tag{13}$$

Substituting (12) and (13) into (10) we obtain the condition for stability in the form

$$\frac{\partial g_1}{\partial \epsilon} < \frac{\dot{\epsilon}}{1+\epsilon}. \tag{14}$$

If the material has a static stress-strain curve, it is given by equating $\dot{\epsilon}$ to zero in (2) or (11). Along this curve, $d\dot{\epsilon} = 0$, giving

$$\frac{\partial g}{\partial \epsilon} = -q\frac{\partial g}{\partial \sigma}, \tag{15}$$

where $q \equiv (\partial\sigma/\partial\epsilon)_{\dot{\epsilon}=0}$, the true static work-hardening rate. Similarly from

(11) we obtain

$$\frac{\partial g_1}{\partial \varepsilon} = -q_1 \frac{\partial g_1}{\partial \sigma_n} , \qquad (16)$$

where $q_1 \equiv (\partial \sigma_n / \partial \varepsilon)_{\dot{\varepsilon}=0}$, the nominal static work-hardening rate.

Equating $\dot{\varepsilon}$ to zero in (10), and combining with (15), gives

$$q > \sigma/(1+\varepsilon) \qquad (17)$$

assuming that $\partial g / \partial \sigma > 0$.

The point on the static stress-strain curve at which $q = \sigma/(1+\varepsilon)$ is given by Considère's construction; thus the deformation remains stable up to this point. Similarly, combining (11) and (16) gives, for $\dot{\varepsilon} = 0$,

$$q_1 > 0, \qquad (18)$$

assuming that $\partial g_1 / \partial \sigma_n > 0$.

The condition (18) shows that for vanishing strain rate, the deformation remains stable up to the point of maximum load, in accordance with the usual simple treatment for rate-independent materials.

III. APPLICATION USING THE 'OVERSTRESS' HYPOTHESIS

Following Sokolovsky (1948) and Malvern (1951) we may assume that the strain rate depends only on the 'overstress', that is the amount by which the stress exceeds the static value for the same strain. Defining the overstress in terms of true stresses, we may then write

$$\dot{\varepsilon} = g(\Delta\sigma) = g[\sigma - f(\varepsilon)], \tag{19}$$

where $\sigma = f(\varepsilon)$ defines the true static stress-strain curve.

It follows from (19) that

$$\frac{\partial g}{\partial \varepsilon} = -g'(\Delta\sigma)f'(\varepsilon), \tag{20}$$

and

$$\frac{\partial g}{\partial \sigma} = g'(\Delta\sigma), \tag{21}$$

where the primes denote differentiation of f and g with respect to their arguments.

Substituting (20) and (21) into (8) and (9) gives

$$P = [f'(\varepsilon) - \frac{\sigma}{1+\varepsilon}]g'(\Delta\sigma) + \frac{\dot{\varepsilon}}{1+\varepsilon} \tag{22}$$

and

$$Q = - \frac{\sigma}{A_o(1+\varepsilon)} \frac{dA_o}{dx} g'(\Delta\sigma). \tag{23}$$

Thus, assuming that $g'(\Delta\sigma) > 0$, the condition for stability of deformation is

$$(1+\varepsilon)f'(\varepsilon) > \sigma - \dot{\varepsilon}/g'(\Delta\sigma). \tag{24}$$

If the overstress is defined in terms of nominal stresses, we write

$$\dot{\varepsilon} = g_1(\Delta\sigma_n) = g_1[\sigma_n - f_1(\varepsilon)], \qquad (25)$$

where $\sigma_n = f_1(\varepsilon)$ defines the nominal static stress-strain curve. The stability condition (14) becomes

$$(1+\varepsilon)f_1'(\varepsilon) > - \dot{\varepsilon}/g_1'(\Delta\sigma_n). \qquad (26)$$

This indicates that the necking strain in a dynamic test is greater than that in a static test, assuming that $g_1'(\Delta\sigma_n) > 0$.

The condition (24) can also be expressed in terms of σ_n and $f_1(\varepsilon)$, by using the relations $f(\varepsilon) = (1+\varepsilon)f_1(\varepsilon)$ and $f'(\varepsilon) = (1+\varepsilon)f_1'(\varepsilon) + f_1(\varepsilon)$. We thus obtain

$$(1+\varepsilon)f_1'(\varepsilon) > \Delta\sigma_n - \dot{\varepsilon}/(1+\varepsilon)g'(\Delta\sigma). \qquad (27)$$

It follows from (27) that the dynamic necking strain is less than the static value if $g'(\Delta\sigma) > g(\Delta\sigma)/\Delta\sigma$, that is, if the strain rate increases more rapidly than the overstress.

It would appear that (19) is a physically more realistic assumption than (25), since the latter implies that for a given true overstress the strain rate decreases with increasing strain. In fact, because of the increasing dislocation density, it is more likely that the strain rate will increase with increasing strain, at a given true overstress. Such an increase would reduce the necking strain below the value given by (27); for example, if we assume that the strain rate is given by $(1+\varepsilon)F(\Delta\sigma)$, we

obtain, instead of (27), the stability condition

$$(1+\epsilon)f_1'(\epsilon) > \Delta\sigma_n. \qquad (28)$$

(28) indicates that the dynamic necking strain is always less than the static value.

IV. APPROXIMATION OF $f(\varepsilon)$ AND $g(\Delta\sigma)$ BY POWER FUNCTIONS

For certain materials, it may be possible to use the empirical expressions

$$f(\varepsilon) = k\varepsilon^n \qquad (29)$$

and

$$g(\Delta\sigma) = C(\Delta\sigma)^p, \qquad (30)$$

to represent the mechanical behavior, k,n,C, and p being constants for a given material. Equation (30) was proposed by Cowper and Symonds (1957), who showed that with $p = 5$ it could represent the results of Manjoine (1944) for the lower yield stress of mild steel. Marsh and Campbell (1963) showed that (30) is also applicable, as a first approximation at least, in the strain hardening region.

The nominal stress-strain curve corresponding to (29) is

$$\sigma_n = f_1(\varepsilon) = k\varepsilon^n/(1+\varepsilon), \qquad (31)$$

and substituting (30) and (31) into (27) we obtain as the condition for stability of deformation

$$\sigma_n < \left(\frac{np}{\varepsilon} - \frac{1}{1+\varepsilon}\right) \frac{k\varepsilon^n}{p-1}. \qquad (32)$$

Figure 1 shows the static stress-strain curve (31), for $n = 1/4$; also plotted in Fig. 1 are lines giving the boundaries of the stable region, defined by (32), for various values of p. It is seen that for $p > 1$, the necking strain is smaller under dynamic loading than the value for static loading.

In general, the static necking strain for a material obeying (31) is given by

$$\varepsilon_s = n/(1-n). \tag{33}$$

If, at the instant when necking begins in a dynamic test, the stress is r times the static value at the same strain, the necking strain given by (32) is

$$\varepsilon_d = np/[r(p-1) - (np-1)]. \tag{34}$$

Comparison of (33) and (34) shows that the effect of raising the strain rate is to reduce the necking strain in the ratio

$$R \equiv \varepsilon_d/\varepsilon_s = (1-N)/(r-N), \tag{35}$$

where

$$N \equiv (np-1)/(p-1). \tag{36}$$

Figure 2 shows R plotted against the fractional increase of stress, (r-1), for various values of N. Equation (35) may be solved for r and used to determine the boundary of the stable region in terms of N and the static stress-strain curve; this gives

$$\sigma_n = f_1(\varepsilon_d)[N + (1-N)\varepsilon_s/\varepsilon_d] \tag{37}$$

or

$$\sigma_n = (\bar{\varepsilon}/\varepsilon_d)f_1(\varepsilon_d) \tag{38}$$

where

$$\bar{\varepsilon} \equiv N\varepsilon_d + (1-N)\varepsilon_s. \tag{39}$$

If N and $f_1(\varepsilon)$ are known, a simple graphical construction may be employed to construct the boundary curve, as shown in Fig. 3. For any given strain ε_d corresponding to a point P on the static stress-strain curve, a point R on the boundary MN is given as follows. The value of $\bar{\varepsilon}$ is first found by dividing the distance between ε_d and ε_s in the ratio $1-N : N$; then the line OP is drawn and produced to meet the line $\varepsilon = \bar{\varepsilon}$ at Q; the point R is then given by drawing a line σ_n = constant through Q to meet the line $\varepsilon = \varepsilon_d$.

V. DISCUSSION AND COMPARISON WITH EXPERIMENTS

It follows from (7) that in the stable region of the (σ,ε) plane, where $P > 0$, the strain gradient λ is at any given instant tending asymptotically towards a value which depends on the initial nonuniformity of the specimen. The rate at which λ tends towards this value is determined largely by the first derivatives of the function $g(\sigma,\varepsilon)$. Thus a strongly rate-dependent material, in which these derivatives are small, will show a less rapid movement towards the limiting value; that is, the stability of such a material is weaker than that of a weakly rate-dependent material. The same argument shows that in the unstable region, the instability is less marked for the highly rate-dependent material. To some extent, this tendency will offset the effect of a reduction in the necking strain as the flow stress is raised, since only in highly rate-dependent materials is the flow stress likely to be raised considerably.

An attempt may be made to relate certain experimental data to the theoretical results derived in Section 4. Stress-strain curves were obtained by Campbell and Cooper (1966) for a low-carbon steel at approximately constant strain rates of 0.001, 0.22, 2, 55 and 106 sec^{-1}, and these curves are plotted in Fig. 4.

In order to apply the results of Section 4, the constants in (29) and (30) must first be determined. We take curve E in Fig. 4 as defining the static nominal stress-strain curve, and this gives the necking strain as $\varepsilon_s = 0.25$; hence from (33) $n = 0.2$. The constant k is adjusted to give the observed static ultimate tensile stress. Equation (31) may then be used to calculate values of σ_n at given values of ε; values so calculated

are shown by filled circles in Fig. 4, and it is seen that they are in good agreement with the experimental curve, for $\varepsilon > 0.1$.

To determine p, values of log ($\Delta\sigma$) are plotted against log $\dot{\varepsilon}$ in Fig. 5; these values correspond to a constant strain of 0.22, this being the estimated mean necking strain in the four dynamic tests. It is seen from Fig. 5 that the rate dependence is adequately approximated by the power law (30): the deviation of the four points from the straight line drawn corresponds to errors of less than $\pm 3\%$ in the measured stresses. The slope of the line gives p = 4.85, but in view of the small number of points available, p will be taken as 5; this coincides with the value adopted by Cowper and Symonds (1957) for mild steel.

Substituting n = 0.2, p = 5 into (36) gives N = 0, and hence from (38) and (39), the boundary of the stable region is given by

$$\sigma_n = (\varepsilon_s/\varepsilon_d)f_1(\varepsilon_d). \qquad (40)$$

Figure 4 shows the curve MN defined by (40). In order to compare this curve with the dynamic test data, it is necessary to determine the value of the slope $d\sigma_n/d\varepsilon$ at which necking starts in a dynamic test. To do this, we write (19) in the form

$$\sigma = f(\varepsilon) + \psi(\dot{\varepsilon}), \qquad (41)$$

where ψ is the inverse of the function g.

Differentiation of (41) gives

$$\frac{d\sigma}{d\varepsilon} = f'(\varepsilon) + \frac{\ddot{\varepsilon}}{\dot{\varepsilon}}\psi'(\dot{\varepsilon}), \qquad (42)$$

where $\psi' = d\psi/d\dot{\varepsilon}$.

Substituting (42) into the stability condition (24) we obtain

$$\frac{d\sigma}{d\varepsilon} - \frac{\sigma}{1+\varepsilon} > \frac{\ddot{\varepsilon} \, \psi'(\dot{\varepsilon})}{\dot{\varepsilon}} - \frac{\dot{\varepsilon}}{(1+\varepsilon)g'(\Delta\sigma)} \, , \tag{43}$$

which becomes, in terms of the nominal stress σ_n,

$$\frac{d\dot{\sigma}_n}{d\varepsilon} > \frac{\ddot{\varepsilon} \, \psi'(\dot{\varepsilon})}{\dot{\varepsilon}(1+\varepsilon)} - \frac{\dot{\varepsilon}}{(1+\varepsilon)^2 g'(\Delta\sigma)} \, . \tag{44}$$

Taking g to be given by (30), the last term in (44) may be written as $-\Delta\sigma_n/p(1+\varepsilon)$; this quantity then defines the slope $d\sigma_n/d\varepsilon$ when necking starts in a dynamic test at constant strain rate. For $p = 5$ and values of $\Delta\sigma_n$ corresponding to the curves of Fig. 4, the slope $-\Delta\sigma_n/p(1+\varepsilon)$ is very small; it may therefore be sufficiently accurately determined by using approximate values of $\Delta\sigma_n$ and ε. The point at which a dynamic stress-strain curve has this slope cannot be obtained very accurately, because of the small curvature; however, the points obtained, shown by open circles in Fig. 4, show reasonable agreement with the theoretical curve MN, with the exception of the point on the curve D. This curve is somewhat anomalous in general: it lies below curve E at strains of about 7% but not at high strains as do curves A, B and C. The reason for these anomalies is unknown.

It may be seen in Fig. 4 that at the highest rates of strain the nominal stress is greater during the Lüders elongation than it is at larger strains. However, the present theory is not applicable during the Lüders elongation, since the strain cannot then be assumed to be uniform

across any given cross-section of the specimen.

Another type of investigation to which the present theory may be
related is that in which rate-dependent materials show very large extensions
when tested in tension at high temperatures. The work of Backofen et al. (1964)
has shown that a near-eutectoid Zn-Al alloy is capable of extensions of more
than 1000% without rupturing, at temperatures in the region of 250°C. The
rate sensitivity of the alloy was measured and found to be very large, and the
observed "superplasticity" was attributed to this.

The static yield stress of the superplastic alloy was shown to be
negligible, the flow stress being given by the equation

$$\sigma = K\dot{\varepsilon}^m, \tag{45}$$

where K and m are constants.

Thus in (19), $f(\varepsilon) = 0$, $g(\Delta\sigma) = B\sigma^\alpha$, with $B = K^{-1/m}$, $\alpha = m^{-1}$;
(22) and (23) then become

$$P = \frac{(1-\alpha)B\sigma^\alpha}{1+\varepsilon} = \frac{(1-\alpha)\dot{\varepsilon}}{1+\varepsilon} \tag{46}$$

and

$$Q = -\frac{\alpha B\sigma^\alpha}{1+\varepsilon} \cdot \frac{1}{A_o}\frac{dA_o}{dx} = -\frac{\alpha\dot{\varepsilon}}{1+\varepsilon}\frac{1}{A_o}\frac{dA_o}{dx}. \tag{47}$$

It follows from (46) that the deformation will be stable if $\alpha < 1$,
i.e. $m > 1$. Backofen et al. showed that, for the alloy they tested, the value
of m varied with strain rate and temperature, reaching values in the region of
0.5 under conditions in which superplasticity was observed. According to (46),
the deformation should be unstable when $m = 0.5$, $\alpha = 2$; however, the rate at

which the strain gradients increase will be very low under these conditions.

As an example, consider a constant strain rate test and assume that $(1/A_o)dA_o/dx = -\mu$, a constant. Then substituting (46) and (47) into (7) we obtain

$$\frac{\partial \lambda}{\partial t} - \frac{(\alpha-1)\dot{\varepsilon}}{1+\dot{\varepsilon}t} \lambda = \frac{\alpha\dot{\varepsilon}\mu}{1+\dot{\varepsilon}t} . \qquad (48)$$

Integrating (48) and assuming that $\lambda = 0$ at $t = 0$, we obtain

$$\lambda = \frac{\alpha\mu}{\alpha-1} [(1+\varepsilon)^{\alpha-1} - 1] . \qquad (49)$$

Equation (49) has been used to obtain the curves of Fig. 6, in which λ/μ is plotted as a function of strain for various values of α. Adopting the value $\lambda/\mu = 10$ as defining a measurable amount of necking, it is seen that if $\alpha = 2$ this condition is only reached at a strain of 500%; in contrast, taking $\alpha = 10$ as typical of a non-superplastic material (Backofen et al. 1964), we find that the condition $\lambda/\mu = 10$ is reached at a strain of about 29%.

The above calculations are based on the assumption that the strain rate is constant everywhere, though this condition cannot be realized when significant necking has occurred. The results show nevertheless that when α is not much greater than unity large extensions are possible without appreciable strain gradients, if μ is small; when α is considerably greater than unity, however, large strain gradients will occur at moderate strains. For example, taking the variation in A_o to be 0.1% along the initial specimen gauge length ℓ_o, so that $\mu\ell_o = 0.001$, we find that for $\alpha = 2$ the strain varies only from 497 to 503% when its mean value is 500%; for $\alpha = 20$, however, when the mean strain is only 30% the variation in strain is from about 20 to 40%.

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CAPTIONS TO FIGURES

1. Stable and unstable plastic flow in a material obeying the constitutive relation $\dot{\varepsilon} = C(\sigma-k\varepsilon^{1/4})^P$.

2. Reduction of necking strain due to rate effect in a material obeying the relation $\dot{\varepsilon} = C(\sigma-k\varepsilon^n)^P$.

3. Graphical construction giving the stability boundary for a material obeying the relation $\dot{\varepsilon} = C(\sigma-k\varepsilon^n)^P$.

4. Experimental dynamic stress-strain curves for mild steel, showing derived stability boundary MN.

5. Logarithmic plot of overstress against strain rate at 22% strain, for mild steel.

6. Curves derived from equation (50), showing the increase of strain gradient λ with increase in strain, for a material obeying equation (46).

$$\sigma_n = \frac{k\epsilon^{1/4}}{1+\epsilon}$$

p = ∞ 5 3 2 1.5 1.2 1

UNSTABLE PLASTIC FLOW

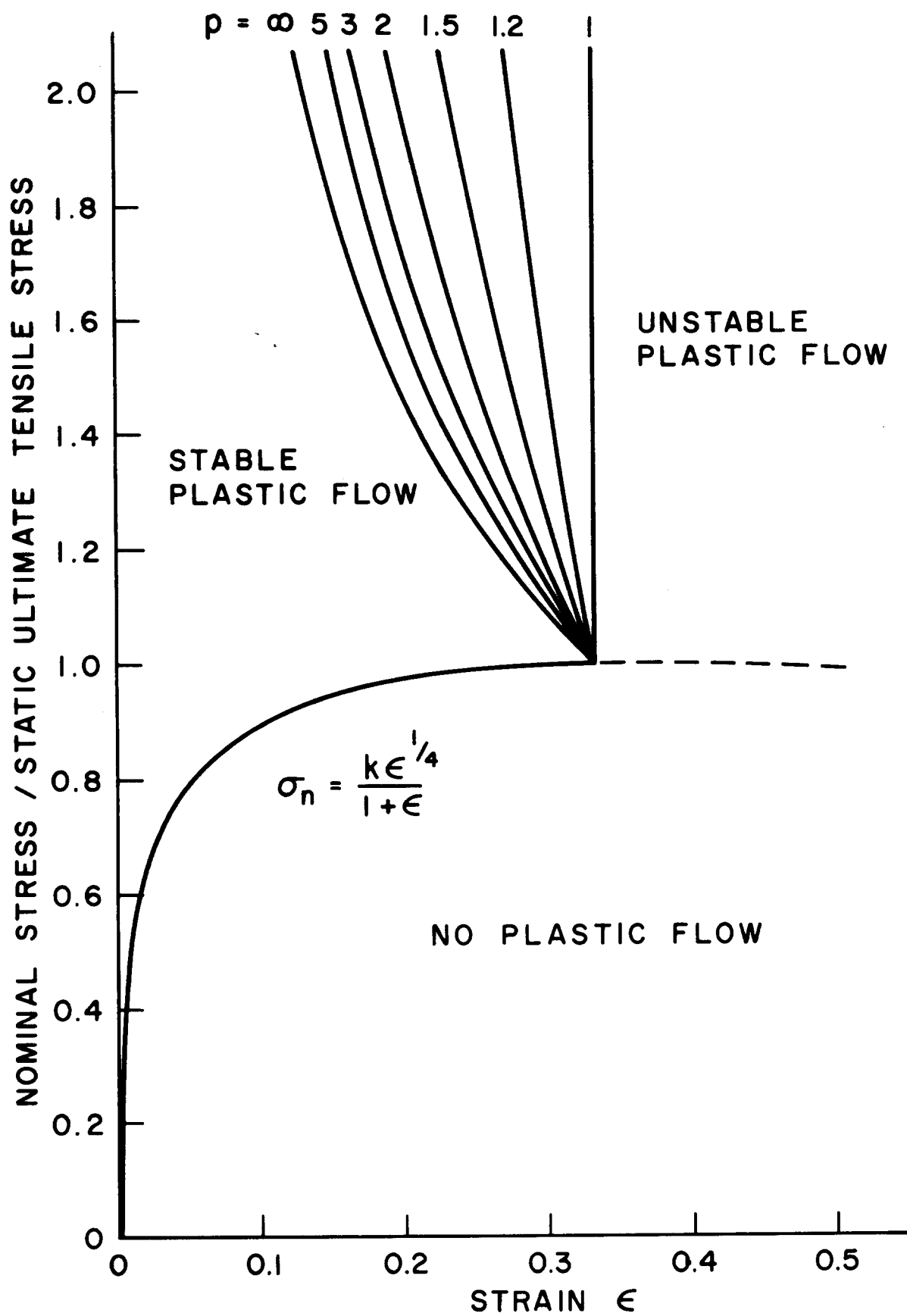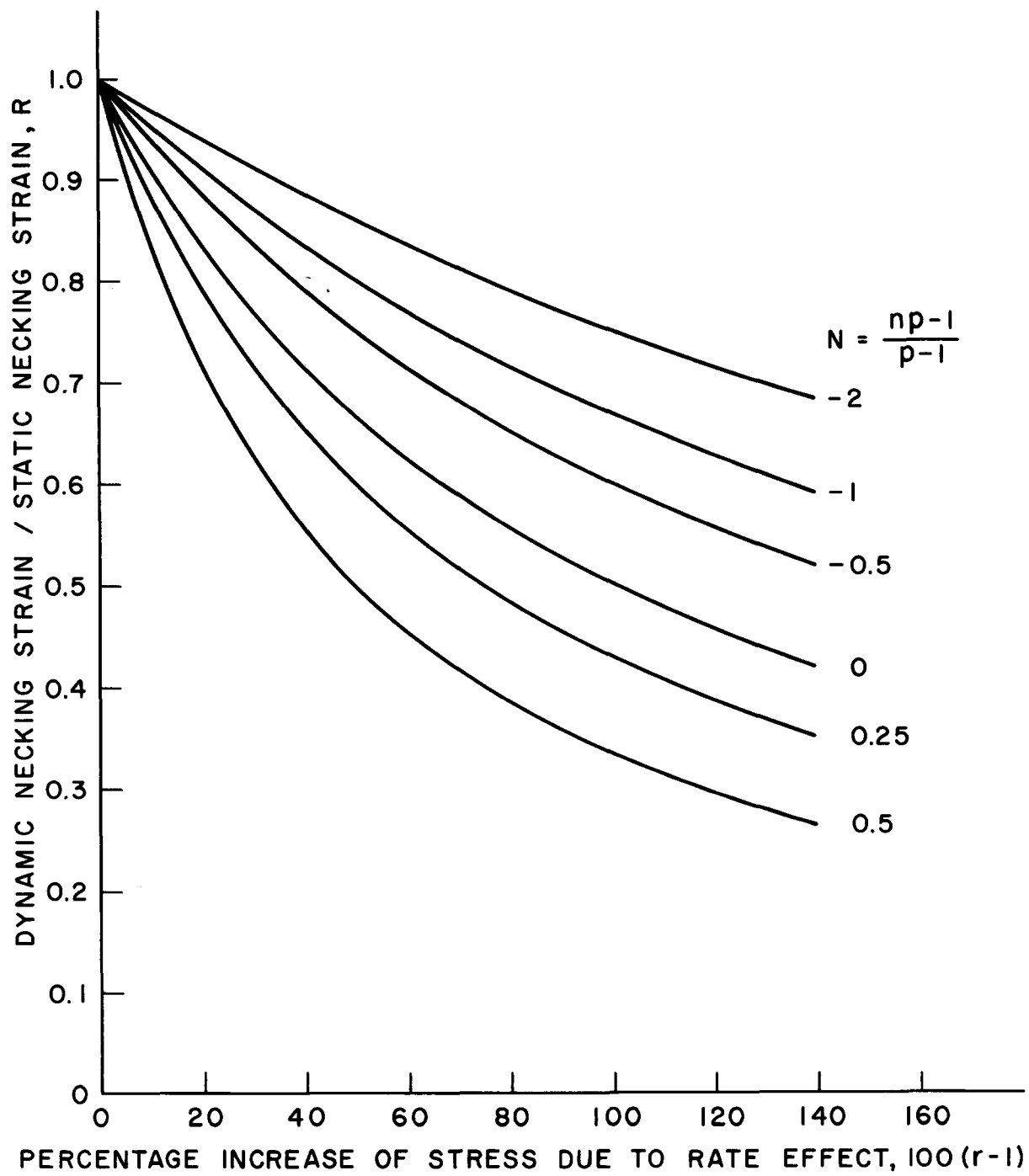STABLE PLASTIC FLOW

NO PLASTIC FLOW

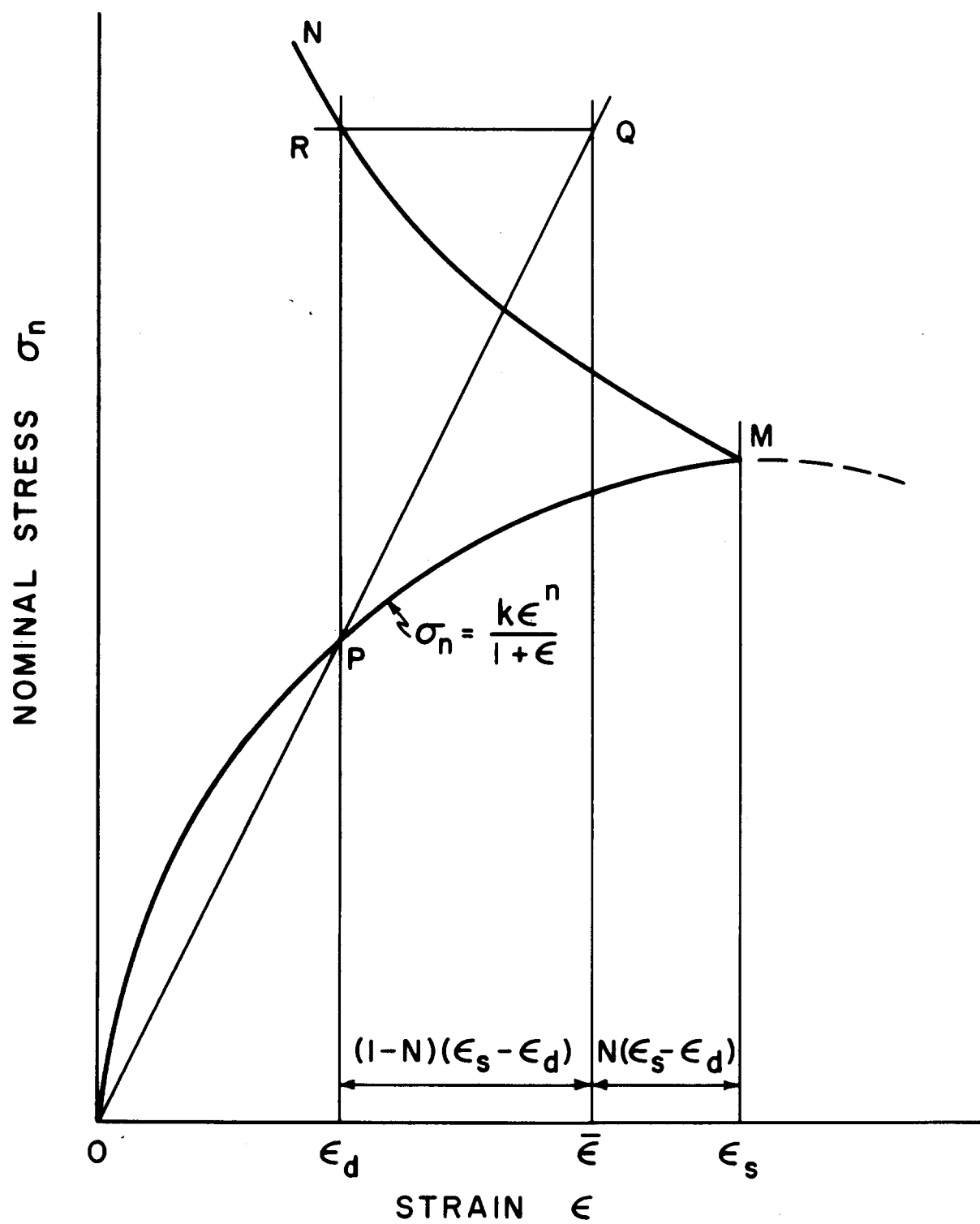NOMINAL STRESS / STATIC ULTIMATE TENSILE STRESS

STRAIN ϵ

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6